Bonds and long-term
Notes payable

Objectives:

Compare bonds versus share financing

Explain the types of bonds and their issuing procedures

Prepare entries to record bonds issued at par

Determine the price of a bonds

Prepare entries to prepare bonds issued at a discount

Prepare entries to record bonds issued at a premium

Record the retirement of bonds

Basics of bonds

A bond is a written promise to pay an amount identified as the par value of the bond along with interest at a stated annual rate.

The par value of the bond, also called the face amount, is paid at a specified future date known as the maturity date.

The stated interest rate, the contract rate, nominal rate, or coupon rate, is quoted as an annual rate.

i.e. $1,000 bond with a contract interest rate of 8% to be paid semi-annually.

The document that specifies the issuer's name, the bonds' par value, the contract interest rate, and the maturity date is called a bond certificate.

Advantages of bonds

1. *Bonds do not affect shareholder control.* Shares reflect an ownership right in the corporation, whereas a bond does not. A *bondholder* has loaned the company money and therefore has a *receivable* from the bond issuer.
2. *Interest on bonds is tax deductible.* Bond interest is tax deductible, but dividends to shareholders are not. To illustrate the importance of this, let's assume a company that pays tax at the rate of 40% issued $1,000,000 of bonds that pay interest at 10%. Interest expense will be $100,000 (= $1,000,000 × 10%). Because interest expense is tax deductible the company's income tax expense will be reduced by the amount of the interest expense times the tax rate or $40,000 (= $100,000 × 40%). Because the corporation saves $40,000 in taxes, the true cost (or after-tax cost) of borrowing is $60,000 (= $100,000 interest expense less $40,000 tax saving). If the same amount of money were raised by issuing shares, instead of bonds, $100,000 paid out as dividends would not be tax deductible and the net costs of raising the $1,000,000 would be the full amount of the dividends.
3. *Bonds can increase return on equity.* Return on equity is net income available to common shareholders divided by common shareholders equity.[1] When a company earns a higher return with the borrowed funds than it is paying in interest, it increases its return on equity. This process is called *financial leverage* or *trading on the equity.*

Illustration

To illustrate the effect on return on equity, let's look at Magnum Skates. Magnum has $1 million in equity P.40, and is planning a $500,000 expansion to meet increasing demand for its product. Magnum predicts the $500,000 expansion will provide $125,000 in additional income before paying any interest. Magnum currently earns $100,000 per year and has no interest expense. Magnum is considering three plans:

- Plan A is to not expand.
- Plan B is to expand, and raise $500,000 from issuing shares.
- Plan C is to sell $500,000 worth of bonds paying 10% annual interest, or $50,000.

Exhibit 17.1 shows us how these three plans affect Magnum's net income, equity, and return on equity (net income ÷ equity).

	Plan A: Do not Expand	Plan B: Increase Equity	Plan C: Issue Bonds
Income before interest expense	$ 100,000	$ 225,000	$ 225,000
Interest expense	—	—	(50,000)
Net income	$ 100,000	$ 225,000	$ 175,000
Equity	$1,000,000	$1,500,000	$1,000,000
Return on equity (net income ÷ equity)	**10.0%**	**15.0%**	**17.5%**

Analysis of these plans shows the corporation will earn a higher return on equity if it expands. The preferred plan of expansion is to issue bonds. Why? Even though the projected net income of $175,000 under Plan C is smaller than the $225,000 under Plan B, the return on equity is larger because of less shareholder investment. This is an important result and proves a general rule: Return on equity increases when the expected rate of return from the new assets is greater than the rate of interest on the bonds. Issuing bonds also allows the current owners to remain in control.

Disadvantages of Bonds

1. *Bonds **require** payment of **both** annual interest and par value at maturity.* Bond payments can be a burden when a company's income is low. Shares, on the other hand, do not require payment of dividends because they are declared at the discretion of the board of directors.
2. *Bonds can decrease return on equity.* When a company earns a lower return with the borrowed funds than it is paying in interest, it decreases its return on equity. This is a risk of bond financing and is more likely to arise when a company has periods of low income.

types of bonds

Types of Bonds	Explanation
1. Secured or Unsecured:	
a. Secured	a. Assets are pledged as a guarantee of payment by the issuing company.
b. Unsecured (called debentures)	b. Backed not by specific assets but only by the earning capacity and credit reputation of the issuer.
2. Term and Serial	
a. Term	a. Principal of all bonds is due in a lump sum at a specified single date.
b. Serial	b. Principal is due in installments at several different dates.
3. Registered and Bearer	
a. Registered	a. Bonds issued registered in the names of the buyers. Ownership records are kept up to date.
b. Bearer	b. Bonds payable to whoever possesses them. No records are kept for change of ownership. Many are coupon bonds meaning that interest is paid to the holder of attached coupons.
4. Convertible, Callable, Redeemable	
a. Convertible	a. Bonds that allow the buyer to exchange the bond for common shares at a fixed ratio.
b. Callable	b. Bonds that may be called for early retirement at the option of the issuing corporation.
c. Redeemable	c. Bonds that may be retired early at the option of the purchaser.

Bond trading

The offering of bonds to the public is called floating an issue.

For convenience, bond market values are expressed as a percent of their face value.

i.e. bond trading at $103^{1/2}$, which means they can be bought for 103.5% of their par value.

The market rate of interest, or effective interest rate, is the amount of interest investors want for lending their money.

Contract rate is:		Bond sells:
Above market rate	➡	At a premium (> 100% of face value)
Equal to market rate	➡	At par value (= 100% of face value)
Below market rate	➡	At a discount (< 100% of face value)

To illustrate an issuance of bonds at par value, let's suppose Barnes Company receives authorization to issue $800,000 of 9%, 20-year bonds. The bonds are dated January 1, 2002, and are due in 20 years on December 31, 2021. They pay interest semi-annually each June 30 and December 31. If all bonds are sold at their par value, Barnes Company makes this entry to record the sale:

2002			
Jan. 1	Cash	800,000	
	Bonds Payable		800,000
	Sold bonds at par.		

This entry reflects increases in the company's cash and long-term liabilities.

Six months later, the first semi-annual interest payment is made, and Barnes records the payment as:

2002			
June 30	Bond Interest Expense	36,000	
	Cash		36,000
	Paid semi-annual interest on bonds; 9% × $800,000 × 6/12.		

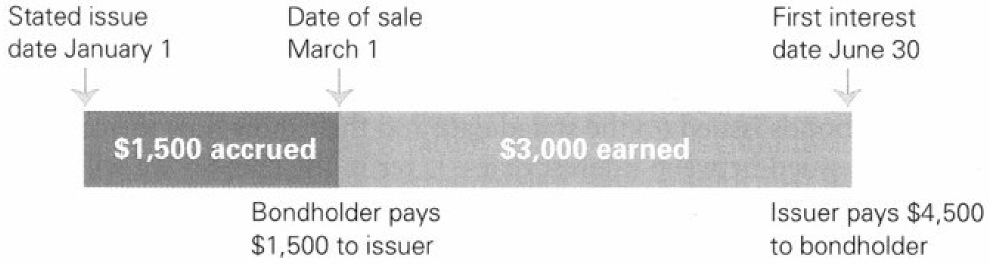
Barnes pays and records the semi-annual interest every six months until the bonds mature.

When the bonds mature 20 years later, Barnes Company records its payment of the maturity value with this entry:

2021			
Jan. 1	Bonds Payable	800,000	
	Cash		800,000
	Paid bonds at maturity.		

Issuing bonds between interest dates

To illustrate, let's suppose that Canadian Tire has $100,000 of 9% bonds available for sale on January 1. Interest is payable semi-annually on each June 30 and December 31. If the bonds are sold at par on March 1, two months after the original issue date of January 1, the issuer collects two months' interest from the buyer at the time of the sale. This amount is $1,500 (= $100,000 × 9% × 2/12) as shown in Exhibit 17.4.



Canadian Tire's entry to record the sale of its bonds on March 1 is:

Mar. 1	Cash	101,500	
	Interest Payable		1,500
	Bonds Payable		100,000
	Sold $100,000 of bonds at par with two months' accrued interest.		

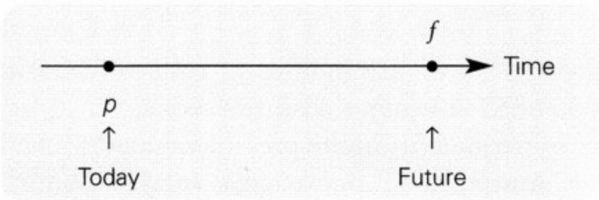
Liabilities for interest payable and the bonds are recorded in separate accounts.

When the June 30 semi-annual interest date arrives, Canadian Tire pays a full six months' interest of $4,500 (= $100,000 × 9% × 6/12) to the bondholder. This payment includes the four months' interest of $3,000 earned by the bondholder from March 1 to June 30 plus the repayment of two months' accrued interest collected by Canadian Tire when the bonds were sold as shown in Exhibit 17.4 above.

Canadian Tire's entry to record this first interest payment is:

June 30	Interest Payable	1,500	
	Bond Interest Expense	3,000	
	Cash		4,500
	Paid semi-annual interest on the bonds.		

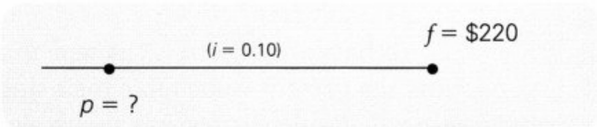
Bond pricing
Present value of a single amount



The formula to compute the present value of this single amount is shown in Exhibit IV.2 where: p = present value; f = future value; i = rate of interest per period; and n = number of periods.

$$p = \frac{f}{(1 + i)^n}$$

To illustrate the application of this formula, let's assume we need $220 one period from today. We want to know how much must be invested now, for one period, at an interest rate of 10% to provide for this $220.[1] For this illustration the p, or present value, is the unknown amount. In particular, the present and future values, along with the interest rate, are shown graphically as:



Conceptually, we know p must be less than $220. This is obvious from the answer to the question: Would we rather have $220 today or $220 at some future date? If we had $220 today, we could invest it and see it grow to something more than $220 in the future. Therefore, if we were promised $220 in the future, we would take less than $220 today. But how much less?

To answer that question we can compute an estimate of the present value of the $220 to be received one period from now using the formula in Exhibit IV.2 as:

$$p = \frac{f}{(1 + i)^n} = \frac{\$220}{(1 + .10)^1} = \$200$$

This means we are indifferent between $200 today or $220 at the end of one period.

We can also use this formula to compute the present value for *any number of periods.* To illustrate this computation, we consider a payment of $242 at the end of two periods at 10% interest. The present value of this $242 to be received two periods from now is computed as:

$$p = \frac{f}{(1 + i)^n} = \frac{\$242}{(1 + .10)^2} = \$200$$

These results tells us we are indifferent between $200 today, or $220 one period from today, or $242 two periods from today.

The number of periods (n) in the present value formula does not have to be expressed in years. Any period of time such as a day, a month, a quarter, or a year can be used. But, whatever period is used, the interest rate (i) must be compounded for the same period. This means if a situation expresses n in months, and i equals 12% per year, then we can assume 1% of an amount invested at the beginning of each month is earned in interest per month and added to the investment. In this case, interest is said to be compounded monthly.

A present value table helps us with present value computations. It gives us present values for a variety of interest rates (i) and a variety of periods (n). Each present value in a present value table assumes the future value (f) is 1. When the future value (f) is different than 1, we can simply multiply present value (p) by that future amount to give us our estimate.

The formula used to construct a table of present values of a single future amount of 1 is shown in Exhibit IV.3.

Case 1 (solve for p when knowing i and n). Our example above is a case in which we need to solve for p when knowing i and n. To illustrate how we use a present value table, let's again look at how we estimate the present value of $220 ($f$) at the end of one period (n) where the interest rate (i) is 10%. To answer this we go to the present value table (Table IV.1) and look in the row for 1 period and in the column for 10% interest. Here we find a present value (p) of 0.9091 based on a future value of 1. This means, for instance, that $1 to be received 1 period from today at 10% interest is worth $0.9091 today. Since the future value is not $1, but is $220, we multiply the 0.9091 by $220 to get an answer of $200.

Case 2 (solve for n when knowing p and i). This is a case in which we have, say, a $100,000 future value ($f$) valued at $13,000 today ($p$) with an interest rate of 12% (i). In this case we want to know how many periods (n) there are between the present value and the future value. A case example is when we want to retire with $100,000, but have only $13,000 earning a 12% return. How long will it be before we can retire? To answer this we go to Table IV.1 and look in the 12% interest column. Here we find a column of present values (p) based on a future value of 1. To use the present value table for this solution, we must divide $13,000 ($p$) by $100,000 ($f$), which equals 0.1300. This is necessary because a present value table defines f equal to 1, and p as a fraction of 1. We look for a value nearest to 0.1300 (p), which we find in the row for 18 periods (n). This means the present value of $100,000 at the end of 18 periods at 12% interest is $13,000 or, alternatively stated, we must work 18 more years.

Case 3 (solve for i when knowing p and n). This is a case where we have, say, a $120,000 future value ($f$) valued at $60,000 ($p$) today when there are nine periods (n) between the present and future values. Here we want to know what rate of interest is being used. As an example, suppose we want to retire with $120,000, but we only have $60,000 and hope to retire in nine years. What interest rate must we earn to retire with $120,000 in nine years? To answer this we go to the present value table (Table IV.1) and look in the row for nine periods. To again use the present value table we must divide $60,000 ($p$) by $120,000 ($f$), which equals 0.5000. Recall this is necessary because a present value table defines f equal to 1, and p as a fraction of 1. We look for a value in the row for nine periods that is nearest to 0.5000 (p), which we find in the column for 8% interest (i). This means the present value of $120,000 at the end of nine periods at 8% interest is $60,000 or, in our example, we must earn 8% annual interest to retire in nine years.

Future Value of a Single Amount

We use the formula for the present value of a single amount and modify it to obtain the formula for the future value of a single amount. To illustrate, we multiply both sides of the equation in Exhibit IV.2 by $(1 + i)^n$. The result is shown in Exhibit IV.4.

$$f = p \times (1 + i)^n$$

Future value (f) is defined in terms of p, i, and n. We can use this formula to determine that \$200 invested for 1 period at an interest rate of 10% increases to a future value of \$220 as follows:

$$\begin{aligned} f &= p \times (1 + i)^n \\ &= \$200 \times (1 + .10)^1 \\ &= \$220 \end{aligned}$$

This formula can also be used to compute the future value of an amount for *any number of periods* into the future. As an example, assume \$200 is invested for three periods at 10%. The future value of this \$200 is \$266.20 and is computed as:

$$\begin{aligned} f &= p \times (1 + i)^n \\ &= \$200 \times (1 + .10)^3 \\ &= \$266.20 \end{aligned}$$

It is also possible to use a future value table to compute future values (f) for many combinations of interest rates (i) and time periods (n). Each future value in a future value table assumes the present value (p) is 1. As with a present value table, if the future amount is something other than 1, we simply multiply our answer by that amount. The formula used to construct a table of future values of a single amount of 1 is shown in Exhibit IV.5.

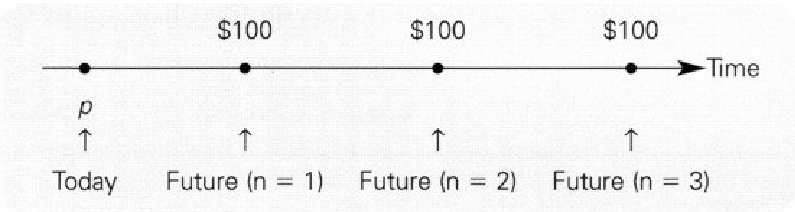
Case 1 (solve for *f* when knowing *i* and *n*). Our example above is a case in which we need to solve for *f* when knowing *i* and *n*. We found that $100 invested for five periods at 12% interest accumulates to $176.24.

Case 2 (solve for *n* when knowing *f* and *i*). This is a case where we have, say, $2,000 (*p*) and we want to know how many periods (*n*) it will take to accumulate to $3,000 (*f*) at 7% (*i*) interest. To answer this, we go to the future value table (Table IV.2) and look in the 7% interest column. Here we find a column of future values (*f*) based on a present value of 1. To use a future value table, we must divide $3,000 (*f*) by $2,000 (*p*), which equals 1.500. This is necessary because a future value table defines *p* equal to 1, and *f* as a multiple of 1. We look for a value nearest to 1.50 (*f*), which we find in the row for six periods (*n*). This means $2,000 invested for six periods at 7% interest accumulates to $3,000.

Case 3 (solve for *i* when knowing *f* and *n*). This is a case where we have, say, $2,001 (*p*) and in nine years (*n*) we want to have $4,000 (*f*). What rate of interest must we earn to accomplish this? To answer this, we go to Table IV.2 and search in the row for nine periods. To use a future value table, we must divide $4,000 (*f*) by $2,001 (*p*), which equals 1.9990. Recall this is necessary because a future value table defines *p* equal to 1, and *f* as a multiple of 1. We look for a value nearest to 1.9990 (*f*), which we find in the column for 8% interest (*i*). This means $2,001 invested for nine periods at 8% interest accumulates to $4,000.

Present Value of an Annuity

An annuity is a series of equal payments occurring at equal intervals. One example is a series of three annual payments of $100 each. The present value of an ordinary annuity is defined as the present value of equal payments at equal intervals as of one period before the first payment. An ordinary annuity of $100 and its present value (p) is illustrated in Exhibit IV.6.



One way for us to compute the present value of an ordinary annuity is to find the present value of each payment using our present value formula from Exhibit IV.3. We then would add up each of the three present values. To illustrate, let's look at three, $100 payments at the end of each of the next three periods with an interest rate of 15%. Our present value computations are:

$$p = \frac{\$100}{(1 + .15)^1} + \frac{\$100}{(1 + .15)^2} + \frac{\$100}{(1 + .15)^3} = \$228.32$$

This computation also is identical to computing the present value of each payment (from Table IV.1) and taking their sum or, alternatively, adding the values from Table IV.1 for each of the three payments and multiplying their sum by the $100 annuity payment.

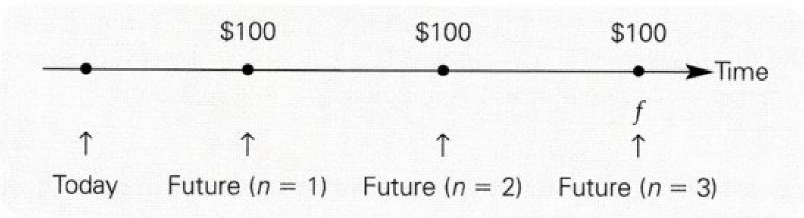
A more direct way is to use a **present value of annuity table**. Table IV.3 at the end of this appendix is one such table. If we look at Table IV.3 where $n = 3$ and $i = 15\%$, we see the present value is 2.2832. This means the present value of an annuity of 1 for 3 periods, with a 15% interest rate, is 2.2832.

A present value of annuity formula is used to construct Table IV.3. It can also be constructed by adding the amounts in a present value of 1 table.[4] To illustrate, we use Tables IV.1 and IV.3 to confirm this relation for the prior example.

From Table IV.1		From Table IV.3	
$i = 15\%, n = 1$	0.8696		
$i = 15\%, n = 2$	0.7561		
$i = 15\%, n = 3$	0.6575		
Total	2.2832	$i = 15\%, n = 3$	2.2832

Future Value of an Annuity

We can also compute the future value of an annuity. The future value of an *ordinary annuity* is the accumulated value of each annuity payment with interest as of the date of the final payment. To illustrate, let's consider the earlier annuity of three annual payments of $100. Exhibit IV.7 shows the point in time for the future value (f). The first payment is made two periods prior to the point where future value is determined, and the final payment occurs on the future value date.



One way to compute the future value of an annuity is to use the formula to find the future value of *each* payment and add them together. If we assume an interest rate of 15%, our calculation is:

$$f = \$100 \times (1 + .15)^2 + \$100 \times (1 + .15)^1 + \$100 \times (1 + .15)^0 = \$347.25$$

This is identical to using Table IV.2 and finding the sum of the future values of each payment, or adding the future values of the three payments of 1 and multiplying the sum by $100.

A more direct way is to use a table showing future values of annuities. Such a table is called a **future value of an annuity of 1 table**. Table IV.4 at the end of this appendix is one such table. We should note in Table IV.4 that when $n = 1$, the future values are equal to 1 ($f = 1$) for all rates of interest. That is because the annuity consists of only one payment and the future value is determined on the date of that payment — no time passes between the payment and its future value.

A formula is used to construct Table IV.4.[5] We can also construct it by adding the amounts from a future value of 1 table. To illustrate, we use Tables IV.2 and IV.4 to confirm this relation for the prior example:

From Table IV.2		From Table IV.4	
$i = 15\%, n = 0$	1.0000		
$i = 15\%, n = 1$	1.1500		
$i = 15\%, n = 2$	1.3225		
Total	3.4725	$i = 15\%, n = 3$	3.4725

Note the future value in Table IV.2 is 1.0000 when $n = 0$, but the future value in Table IV.4 is 1.0000 when $n = 1$. Is this a contradiction? No. When $n = 0$ in Table IV.2, the future value is determined on the date where a single payment occurs. This means no interest is earned, since no time has passed, and the future value equals the payment. Table IV.4 describes annuities with equal payments occurring at the ***end*** of each period. When $n = 1$, the annuity has one payment, and its future value equals 1 on the date of its final and only payment. Again, no time passes from the payment and its future value date.

Problems

You are asked to make future value estimates using the future value of 1 table (Table IV.2). Which interest rate column do you use when working with the following rates?

a. 8% compounded quarterly
b. 12% compounded annually
c. 6% compounded semiannually
d. 12% compounded monthly

Flaherty is considering an investment that, if paid for immediately, is expected to return $140,000 five years hence. If Flaherty demands a 9% return, how much is she willing to pay for this investment?

CII, Inc., invested $630,000 in a project expected to earn a 12% annual rate of return. The earnings will be reinvested in the project each year until the entire investment is liquidated 10 years hence. What will the cash proceeds be when the project is liquidated?

Beene Distributing is considering a contract that will return $150,000 annually at the end of each year for six years. If Beene demands an annual return of 7% and pays for the investment immediately, how much should it be willing to pay?

Claire Fitch is planning to begin an individual retirement program in which she will invest $1,500 annually at the end of each year. Fitch plans to retire after making 30 annual investments in a program that earns a return of 10%. What will be the value of the program on the date of the last investment?

Ken Francis has been offered the possibility of investing $2,745 for 15 years, after which he will be paid $10,000. What annual rate of interest will Francis earn? (Use Table IV.1.)

Megan Brink has been offered the possibility of investing $6,651. The investment will earn 6% per year and will return Brink $10,000 at the end of the investment. How many years must Brink wait to receive the $10,000? (Use Table IV.1.)

For each of the following situations identify (1) it as either (a) present or future value and (b) single amount or annuity case, (2) the table you would use in your computations (but do not solve the problem), and (3) the interest rate and time periods you would use.

a. You need to accumulate $10,000 for a trip you wish to take in four years. You are able to earn 8% compounded semiannually on your savings. You only plan on making one deposit and letting the money accumulate for four years. How would you determine the amount of the one-time deposit?

b. Assume the same facts as in (a), except you will make semiannual deposits to your savings account.

c. You hope to retire after working 40 years with savings in excess of $1,000,000. You expect to save $4,000 a year for 40 years and earn an annual rate of interest of 8%. Will you be able to retire with more than $1,000,000 in 40 years?

d. A sweepstakes agency names you a grand prize winner. You can take $225,000 immediately or elect to receive annual installments of $30,000 for 20 years. You can earn 10% annually on investments you make. Which prize do you choose to receive?

Bill Thompson expects to invest $10,000 at 12% and, at the end of the investment, receive $96,463. How many years will elapse before Thompson receives the payment? (Use Table IV.2.)

Ed Summers expects to invest $10,000 for 25 years, after which he will receive $108,347. What rate of interest will Summers earn? (Use Table IV.2.)

Betsey Jones expects an immediate investment of $57,466 to return $10,000 annually for eight years, with the first payment to be received in one year. What rate of interest will Jones earn? (Use Table IV.3.)

Table IV.1

Present Value of 1 Due in n Periods

	Rate											
Periods	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	15%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.8929	0.8696
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264	0.7972	0.7561
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7118	0.6575
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6355	0.5718
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5674	0.4972
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645	0.5066	0.4323
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132	0.4523	0.3759
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665	0.4039	0.3269
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241	0.3606	0.2843
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855	0.3220	0.2472
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505	0.2875	0.2149
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186	0.2567	0.1869
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897	0.2292	0.1625
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633	0.2046	0.1413
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394	0.1827	0.1229
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176	0.1631	0.1069
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978	0.1456	0.0929
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799	0.1300	0.0808
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635	0.1161	0.0703
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486	0.1037	0.0611
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923	0.0588	0.0304
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573	0.0334	0.0151
35	0.7059	0.5000	0.3554	0.2534	0.1813	0.1301	0.0937	0.0676	0.0490	0.0356	0.0189	0.0075
40	0.6717	0.4529	0.3066	0.2083	0.1420	0.0972	0.0668	0.0460	0.0318	0.0221	0.0107	0.0037

Table IV.2

Future Value of 1 Due in *n* Periods

	Rate											
Periods	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	15%
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1200	1.1500
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1811	1.2100	1.2544	1.3225
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.4049	1.5209
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5735	1.7490
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.7623	2.0114
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7116	1.9738	2.3131
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.2107	2.6600
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.4760	3.0590
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.7731	3.5179
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	3.1058	4.0456
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531	3.4785	4.6524
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384	3.8960	5.3503
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523	4.3635	6.1528
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975	4.8871	7.0757
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772	5.4736	8.1371
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950	6.1304	9.3576
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545	6.8660	10.7613
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599	7.6900	12.3755
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159	8.6128	14.2318
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275	9.6463	16.3665
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.8347	17.0001	32.9190
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.0627	13.2677	17.4494	29.9599	66.2118
35	1.4166	1.9999	2.8139	3.9461	5.5160	7.6861	10.6766	14.7853	20.4140	28.1024	52.7996	133.176
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.2857	14.9745	21.7245	31.4094	45.2593	93.0510	267.864

Table IV.3

Present Value of an Annuity of 1 per Period

Periods	Rate 1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	15%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.8929	0.8696
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	1.6901	1.6257
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869	2.4018	2.2832
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.0373	2.8550
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6048	3.3522
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.1114	3.7845
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.5638	4.1604
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	4.9676	4.4873
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7950	5.3282	4.7716
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.6502	5.0188
11	10.3676	9.7868	9.2526	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951	5.9377	5.2337
12	11.2551	10.5753	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137	6.1944	5.4206
13	12.1337	11.3484	10.6350	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034	6.4235	5.5831
14	13.0037	12.1062	11.2961	10.5631	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667	6.6282	5.7245
15	13.8651	12.8493	11.9379	11.1184	10.3797	9.7122	9.1079	8.5595	8.0607	7.6061	6.8109	5.8474
16	14.7179	13.5777	12.5611	11.6523	10.8378	10.1059	9.4466	8.8514	8.3126	7.8237	6.9740	5.9542
17	15.5623	14.2919	13.1661	12.1657	11.2741	10.4773	9.7632	9.1216	8.5436	8.0216	7.1196	6.0472
18	16.3983	14.9920	13.7535	12.6593	11.6896	10.8276	10.0591	9.3719	8.7556	8.2014	7.2497	6.1280
19	17.2260	15.6785	14.3238	13.1339	12.0853	11.1581	10.3356	9.6036	8.9501	8.3649	7.3658	6.1982
20	18.0456	16.3514	14.8775	13.5903	12.4622	11.4699	10.5940	9.8181	9.1285	8.5136	7.4694	6.2593
25	22.0232	19.5235	17.4131	15.6221	14.0939	12.7834	11.6536	10.6748	9.8226	9.0770	7.8431	6.4641
30	25.8077	22.3965	19.6004	17.2920	15.3725	13.7648	12.4090	11.2578	10.2737	9.4269	8.0552	6.5660
35	29.4086	24.9986	21.4872	18.6646	16.3742	14.4982	12.9477	11.6546	10.5668	9.6442	8.1755	6.6166
40	32.8347	27.3555	23.1148	19.7928	17.1591	15.0463	13.3317	11.9246	10.7574	9.7791	8.2438	6.6418

Table IV.4

Future Value of an Annuity of 1 per Period

Periods	Rate 1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	15%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1200	2.1500
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3744	3.4725
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7793	4.9934
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.3528	6.7424
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	8.1152	8.7537
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	10.0890	11.0668
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359	12.2997	13.7268
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795	14.7757	16.7858
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374	17.5487	20.3037
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	15.7835	16.6455	17.5603	18.5312	20.6546	24.3493
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771	20.1407	21.3843	24.1331	29.0017
13	13.8093	14.6803	15.6178	16.6268	17.7130	18.8821	20.1406	21.4953	22.9534	24.5227	28.0291	34.3519
14	14.9474	15.9739	17.0863	18.2919	19.5986	21.0151	22.5505	24.2149	26.0192	27.9750	32.3926	40.5047
15	16.0969	17.2934	18.5989	20.0236	21.5786	23.2760	25.1290	27.1521	29.3609	31.7725	37.2797	47.5804
16	17.2579	18.6393	20.1569	21.8245	23.6575	25.6725	27.8881	30.3243	33.0034	35.9497	42.7533	55.7175
17	18.4304	20.012	21.7616	23.6975	25.8404	28.2129	30.8402	33.7502	36.9737	40.5447	48.8837	65.0751
18	19.6147	21.4123	23.4144	25.6454	28.1324	30.9057	33.9990	37.4502	41.3013	45.5992	55.7497	75.8364
19	20.8109	22.8406	25.1169	27.6712	30.5390	33.7600	37.3790	41.4463	46.0185	41.1591	63.4397	88.2118
20	22.0190	24.2974	26.8704	29.7781	33.0660	36.7856	40.9955	45.7620	51.1601	57.2750	72.0524	102.444
25	28.2432	32.0303	36.4593	41.6459	47.7271	54.8645	63.2490	73.1059	84.7009	98.3471	133.334	212.793
30	34.7849	40.5681	47.5754	56.0849	66.4388	79.0582	94.4608	113.283	136.308	164.494	241.333	434.745
35	41.6603	49.9945	60.4621	73.6522	90.3203	111.435	138.237	172.317	215.711	271.024	431.663	881.170
40	48.8864	60.4020	75.4013	95.0255	120.800	154.762	199.635	259.057	337.882	442.593	767.091	1,779.09