LEARNING OBJECTIVES

After studying this chapter, you should be able to:

1. Distinguish between the general case and a special case of CVP
2. Explain the relationship between operating income and net income
3. Describe the assumptions underlying CVP
4. Demonstrate three methods for determining the breakeven point and target operating income
5. Explain how sensitivity analysis can help managers cope with uncertainty
6. Illustrate how CVP can assist cost planning
7. Describe the effect of revenue mix on operating income
8. Illustrate how CVP can incorporate income taxes

Revenue driver and cost drivers:

Cost-volume-profit (CVP) analysis examines the behaviour of total revenues, total costs, and operating income as changes occur in the output level, selling price, variable costs, or fixed costs.

A revenue driver is a factor that affects revenues.i.e. Units of output sold, selling prices, and levels of marketing costs.

Total costs = variable costs + fixed costs

Total revenues – total costs = operating income

operating income – income taxes = net income

Example of output:

Industry	Measure of Output
Airlines	Passenger-miles
Automobiles	Vehicles manufactured
Hospitals	Patient-days
Hotels/motels	Rooms occupied
Universities	Student credit-hours

The following abbreviations are used in this chapter:

- USP = Unit selling price
- UVC = Unit variable costs
- UCM = Unit contribution margin (USP – UVC)
- FC = Fixed costs
- Q = Quantity of output units sold (or manufactured)
- OI = Operating income
- TOI = Target operating income
- NI = Net income

A CVP analysis is based on the following assumptions:

1. Total costs can be divided into a fixed component and a component that varies with the level of output.
2. The behaviour of total revenues and total costs is linear in relation to output units within the relevant range.
3. The unit selling price, unit variable costs, and fixed costs are known.

The breakeven point

The breakeven point is the quantity of output where total revenues and total costs are equal—where operating incime is zero.

Example: Mary Frost plans to sell Do-All Software, a software package, at a heavily attended two-day computer convention in Montreal. Mary can purchase this software from a computer software wholesaler at $120 per package with the privilege of returning all unsold units and receiving a full $120 rebate per package. The units (packages) will be sold at $200 each. Frost has already paid $2,000 to Computer Conventions, Inc. for the booth rental for the two-day convention. What quantity of units will she need to sell in order to break even? Assume there are no other costs.

Equation Method

The first approach for computing the breakeven point is the equation method. Using the terminology in this chapter, the income statement can be expressed in equation form as follows:

$$\text{Revenues} - \text{Variable costs} - \text{Fixed costs} = \text{Operating income}$$

$$(\text{USP} \times \text{Q}) - (\text{UVC} \times \text{Q}) - \text{FC} = \text{OI}$$

This equation provides the most general and easy-to-remember approach to any CVP situation. Setting operating income equal to zero in the preceding equation, we obtain:

$$\$200\text{Q} - \$120\text{Q} - \$2{,}000 = \$0$$

$$\$80\text{Q} = \$2{,}000$$

$$\text{Q} = \$2{,}000 \div \$80 = 25 \text{ units}$$

If Frost sells fewer than 25 units, she will have a loss; if she sells 25 units, she will break even; and if she sells more than 25 units, she will make a profit. This breakeven point is expressed in units. It can also be expressed in sales dollars: 25 units × $200 selling price = $5,000.

Contribution margin method

Contribution margin is equal to revenues minus all costs or the outputthat vary with respect to the units of output.

$$(USP \times Q) - (UVC \times Q) - FC = OI$$

$$(USP - UVC) \times Q = FC + OI$$

$$UCM \times Q = FC + OI$$

$$Q = \frac{FC + OI}{UCM}$$

At the breakeven point, operating income is, by definition, zero. Setting OI = 0, we obtain:

$$\text{Breakeven number of units} = \frac{\text{Fixed costs}}{\text{Unit contribution margin}}$$

$$= \frac{FC}{UCM}$$

The calculations in the equation method and the contribution margin method appear similar because one is merely a restatement of the other. In our example, fixed costs are $2,000 and the unit contribution margin is $80 ($200 – $120). Therefore:

$$\text{Breakeven number of units} = \$2{,}000 \div \$80 = 25 \text{ units}$$

A **contribution income statement** groups line items by cost behaviour pattern to highlight the contribution margin. The following such statement confirms the preceding breakeven calculations:

Revenues, $200 × 25	$5,000
Variable costs, $120 × 25	3,000
Contribution margin, $80 × 25	2,000
Fixed costs	2,000
Operating income	$ 0

Graph Method

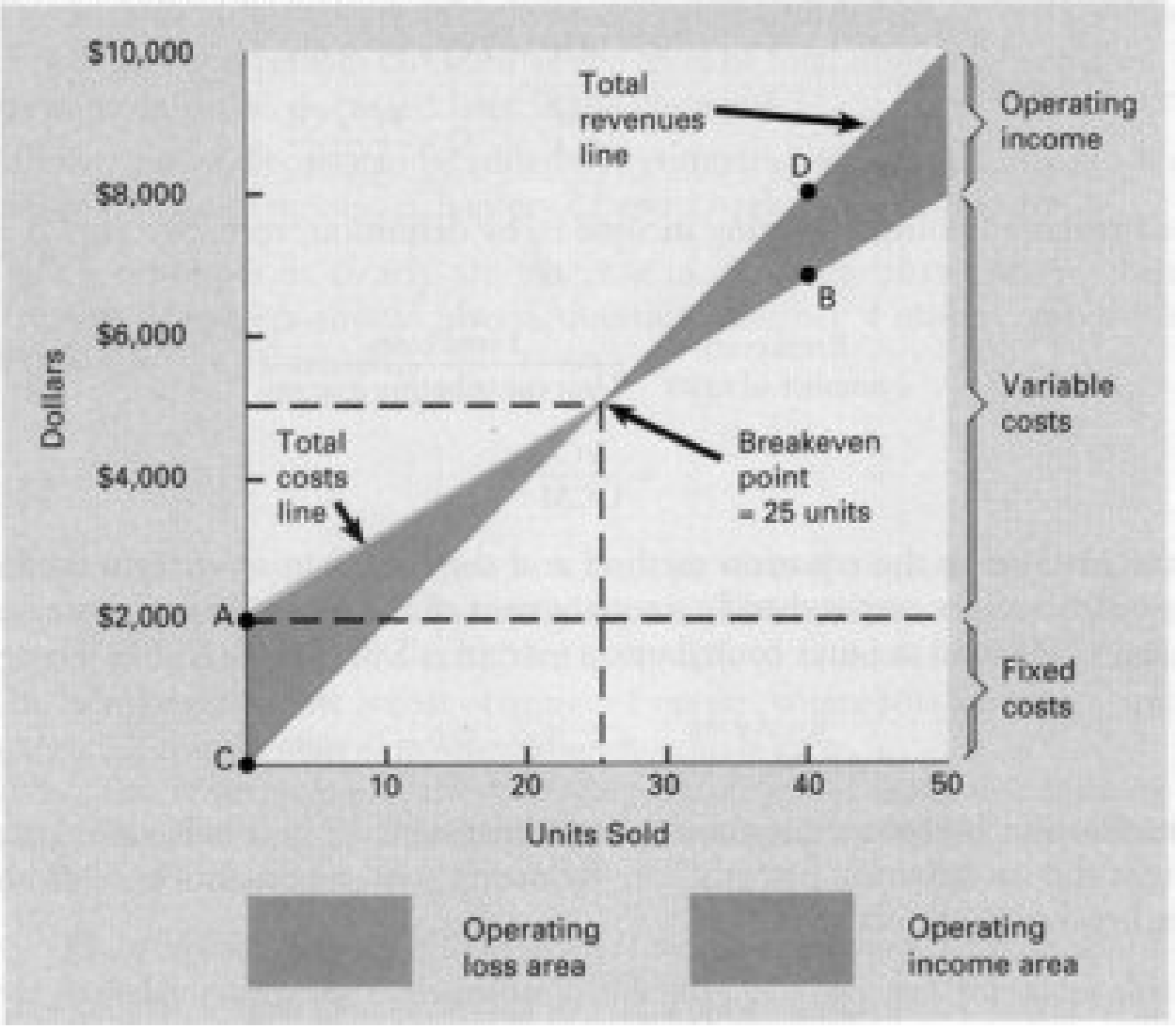
In the graph method, we plot the total costs line and the total revenues line. Their point of intersection is the breakeven point. Exhibit 3-1 illustrates this method for our Do-All example. We need only two points to plot each line if each is assumed to be linear:

1. **Total costs line.** This line is the sum of the fixed costs and the variable costs. Fixed costs are $2,000 at all output levels within the relevant range. To plot fixed costs, measure $2,000 on the vertical axis (point A) and extend a line horizontally. Variable costs are $120 per unit. To plot the total costs line, use as one point the $2,000 fixed costs at 0 output units (point A). Select a second point by choosing any other convenient output level (say, 40 units) and determining the corresponding total costs. The total variable costs at this output level are $4,800 (40 × $120). Fixed costs are $2,000 at all output levels within the relevant range. Hence, total costs at 40 units of output are $6,800, which is point B in Exhibit 3-1. The total costs line is the straight line from point A passing through point B.

2. **Total revenues line.** One convenient starting point is zero revenues at the zero output level, which is point C in Exhibit 3-1. Select a second point by choosing any other convenient output level and determining its total revenues. At 40 units of output, total revenues are $8,000 (40 × $200), which is point D in Exhibit 3-1. The total revenues line is the straight line from point C passing through point D.

The breakeven point is where the total revenues line and the total costs line intersect. At this point, total revenues equal total costs. But Exhibit 3-1 shows the profit or loss outlook for a wide range of output levels. Many people describe the topics covered in this chapter as *breakeven analysis*. We prefer to use the phrase *cost-volume-profit analysis* to avoid overemphasizing the single point where total revenues equal total costs. Managers want to know how operating income differs at many different output levels.

EXHIBIT 3-1
Cost-Volume-Profit Graph



Target Operating Income

Let us introduce a profit element by asking: How many units must be sold to earn an operating income of $1,200? The equation method provides a straightforward way to answer this question:

$$\text{Let QT} = \text{Number of units sold to earn target operating income}$$

$$\text{Revenues} - \text{Variable costs} - \text{Fixed costs} = \text{Target operating income}$$

$$\$200\text{QT} - \$120\text{QT} - \$2{,}000 = \$1{,}200$$

$$\$80\text{QT} = \$2{,}000 + \$1{,}200$$

$$\$80\text{QT} = \$3{,}200$$

$$\text{QT} = \$3{,}200 \div \$80 = 40 \text{ units}$$

Alternatively, we could use the contribution margin method. The numerator now consists of fixed costs plus target operating income:

$$\text{QT} = \frac{\text{Fixed costs} + \text{Target operating income}}{\text{Unit contribution margin}} = \frac{\text{FC} + \text{TOI}}{\text{UCM}}$$

$$\text{QT} = \frac{\$2{,}000 + \$1{,}200}{\$80}$$

$$\$80\text{QT} = \$3{,}200$$

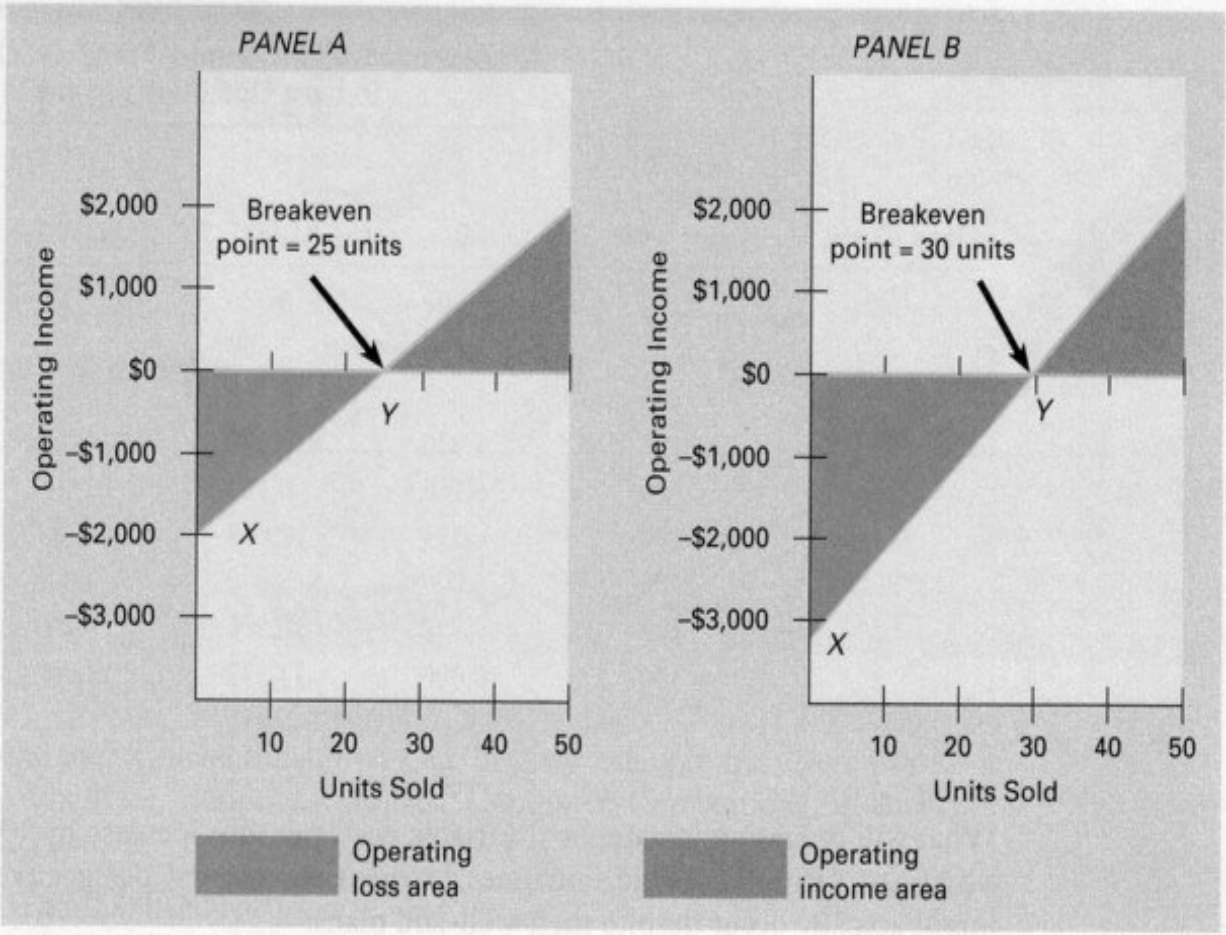
$$\text{QT} = \$3{,}200 \div \$80 = 40 \text{ units}$$

Proof:

Revenues, $200 × 40	$8,000
Variable costs, $120 × 40	4,800
Contribution margin, $80 × 40	3,200
Fixed costs	2,000
Operating income	$1,200

The graph in Exhibit 3-1 indicates that at the 40-unit output level, the difference between total revenues and total costs is the $1,200 operating income.

EXHIBIT 3-2
The Profit-Volume Graph



We can recast Exhibit 3-1 in the form of a profit-volume (PV) graph. A **PV graph** shows the impact on operating income of changes in the output level. Exhibit 3-2 (Panel A) presents the PV graph for Do-All (fixed costs of $2,000, selling price of $200, and variable costs per unit of $120). The PV line can be drawn using two points. One convenient point (X) is the level of fixed costs at zero output—$2,000, which is also the operating loss at this output level. A second convenient point (Y) is the breakeven point—25 units in our example (see p. 60). The PV line is drawn by connecting points X and Y and extending the line beyond Y. Each unit sold beyond the breakeven point will add $80 to operating income. At the 35-unit output level, for example, operating income would be $800:

$$(\$200 \times 35) - (\$120 \times 35) - \$2{,}000 = \$800$$

A comparison of PV charts representing different what-if possibilities can highlight their effects on operating income. Panel B in Exhibit 3-2 shows the PV chart for Do-All assuming fixed costs of $3,300 (compared with $2,000 in Panel A) and variable costs per unit of $90 (compared with $120 in Panel A). The selling price is $200 in both graphs. The unit contribution margin in Panel B is $110. The breakeven point in Panel B is 30 units:

$$\$200Q - \$90Q - \$3{,}300 = 0$$

$$Q = \$3{,}300 \div \$110 = 30 \text{ units}$$

Each unit sold beyond the breakeven point will add $110 to operating income. The PV graph in Panel B has a steeper slope for its operating income line, which means that the operating income increases at a faster rate as the level of output increases.

Sensitivity analysis

What-if analysis: What will operating income be if the output level decreases?What will operating be if variable costs per unit increase?

Spreadsheet Analysis of CVP Relationships for Do-All Software

Fixed Costs	Variable Costs per Unit	Revenue Dollars Required at $200 Selling Price to Earn Operating Income Of: $0	$1,000	$1,500	$2,000
$2,000	$100	$ 4,000	$ 6,000	$ 7,000	$ 8,000
	120	5,000	7,500	8,750	10,000
	140	6,667	10,000	11,667	13,333
$2,500	$100	$ 5,000	$ 7,000	$ 8,000	$ 9,000
	120	6,250	8,750	10,000	11,250
	140	8,333	11,667	13,333	15,000
$3,000	$100	$ 6,000	$ 8,000	$ 9,000	$10,000
	120	7,500	10,000	11,250	12,500
	140	10,000	13,333	15,000	16,667

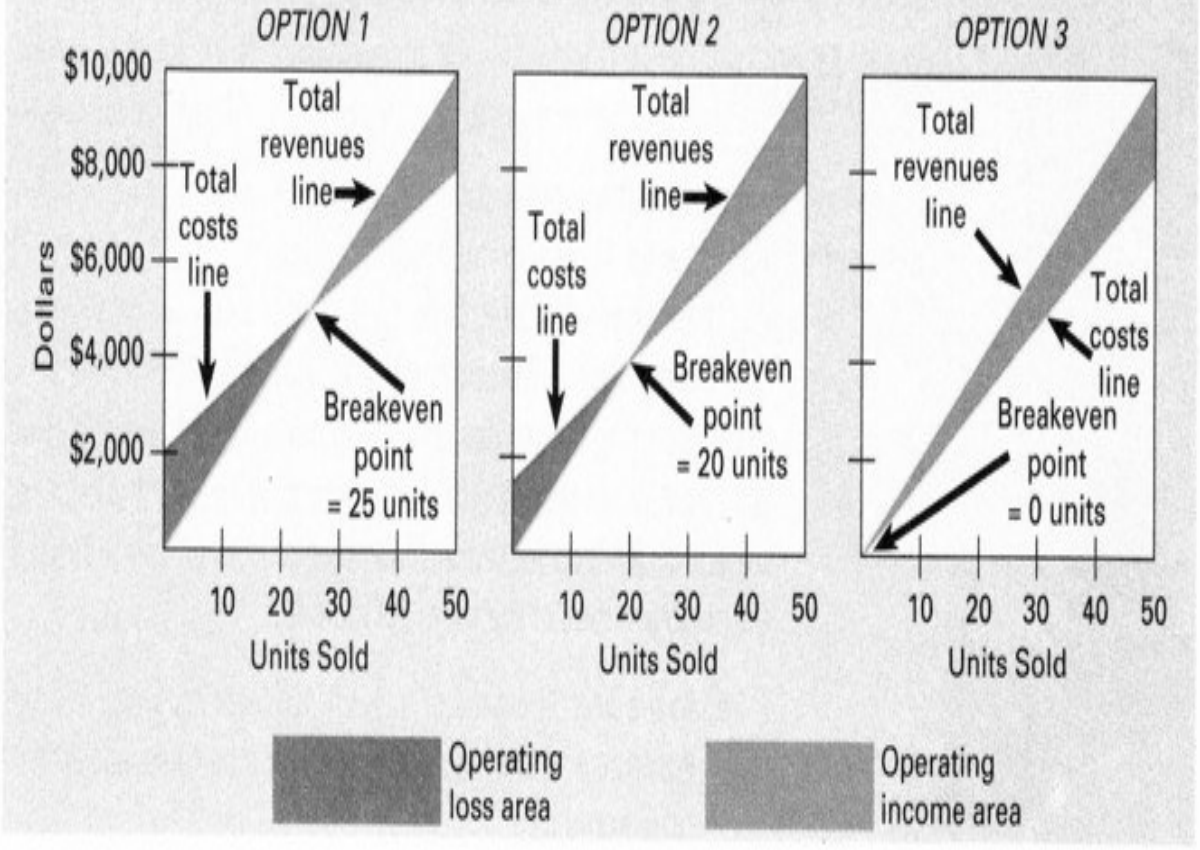
The widespread use of electronic spreadsheets has promoted the use of CVP analysis in many organizations. Using spreadsheets, managers can easily conduct CVP-based sensitivity analyses to examine the effect and interaction of changes in selling prices, unit variable costs, fixed costs, and target operating incomes. Exhibit 3-3 displays a spreadsheet for our Do-All example. Mary Frost can immediately see the revenues that need to be generated to reach particular operating income levels, given alternative levels of fixed costs and variable costs per unit. For example, revenues of $6,000 (30 units at $200 per unit) are required to earn an operating income of $1,000 if fixed costs are $2,000 and variable costs per unit are $100. Frost can also use Exhibit 3-3 to assess whether she wants to sell at the Montreal computer convention if, for example, the booth rental is raised to $3,000 (thus increasing fixed costs to $3,000) or the software supplier raises its price to $140 per unit (thus increasing variable costs to $140 per unit).

One aspect of sensitivity analysis is the **margin of safety,** which is the excess of budgeted revenues over the breakeven revenues. The margin of safety is the answer to the what-if question: If budgeted revenues are above breakeven and drop, how far can they fall below budget before the breakeven point is reached? Such a fall could be due to a competitor having a better product, poorly executed marketing, and so on. Assume that Mary Frost has fixed costs of $3,000, a selling price of $200, and variable costs per unit of $140. For 75 units sold, the budgeted revenues are $15,000 and the budgeted operating income is $1,500. The breakeven point for this set of assumptions is 50 units ($3,000 ÷ $60) or $10,000 ($200 × 50). Hence, the margin of safety is $5,000 ($15,000 – $10,000) or 25 units.

Alternative fixed-cost/variablecost structure
What would happen if Computer conventions offers Frost three rental alternatives.

- Option 1. $2,000 fixed fee
- Option 2. $1,400 fixed fee plus 5% of the convention revenues from Do-All sales
- Option 3. 20% of the convention revenues from Do-All sales with no fixed fee

CVP Graphs for Alternative Rental Schedules for Do-All Software



- **Option 1** exposes her to fixed costs of $2,000 and a breakeven point of 25 units. This option brings $80 additional operating income for each unit sold above 25 units.
- **Option 2** exposes her to lower fixed costs of $1,400 and a lower breakeven point of 20 units. There is, however, only $70 in additional operating income for each unit sold above 20 units.
- **Option 3** has no fixed costs. Frost makes $40 in additional operating income for each unit sold. This $40 addition to operating income starts from the first unit sold. This option enables Frost to break even if no units are sold.[2]

CVP analysis highlights the different risks and different returns associated with each option. For example, while option 1 has the most downside risk (a $2,000 fixed up-front payment), it also has the highest contribution margin per unit. This $80 contribution margin per unit translates to high upside potential if Frost is able to generate sales above 25 units. By moving from option 1 to option 2, Frost faces less risk (lowers her fixed costs) if demand is low, but she must accept less upside potential (because of the higher variable costs) if demand is high. The choice between options 1, 2, and 3 will be influenced by her confidence in the level of demand for Do-All software and her willingness to risk money.

Revenue mix (also called **sales mix**) is the relative combination of quantities of products or services that constitutes total revenues. If the mix changes, overall revenue targets may still be achieved. However, the effects on operating income depend on how the original proportions of lower or higher contribution margin products have shifted.

Suppose Mary Frost in our computer convention example is now budgeting for the next convention. She plans to sell two software products—Do-All and Superword—and budgets the following:

	Do-All	Superword	Total
Units sold	60	30	90
Revenues, $200 and $130 per unit	$12,000	$ 3,900	$15,900
Variable costs, $120 and $90 per unit	7,200	2,700	9,900
Contribution margin, $80 and $40 per unit	$ 4,800	$ 1,200	6,000
Fixed costs			2,000
Operating income			$ 4,000

What is the breakeven point? Unlike the single product (or service) situation, there is not a unique number of units for a multiple-product situation. This number instead depends on the revenue mix. The following approach can be used when it is assumed that the budgeted revenue mix (two units of Do-All sold for each unit of Superword sold) will not change at different levels of total revenue:

$$\text{Let } S = \text{Number of units of Superword to break even}$$

$$2S = \text{Number of units of Do-All to break even}$$

$$\text{Revenues} - \text{Variable costs} - \text{Fixed costs} = \text{Operating income}$$

$$[\$200(2S) + \$130S] - [\$120(2S) + \$90S] - \$2{,}000 = 0$$

$$\$530S - \$330S = \$2{,}000$$

$$\$200S = \$2{,}000$$

$$S = 10$$

$$2S = 20$$

The breakeven point is 30 units when the revenue mix is 20 units of Do-All and 10 units of Superword. The total contribution margin of $2,000 (Do-All $80 × 20 = $1,600 plus Superword $40 × 10 = $400) equals the fixed costs of $2,000 at this mix.

Alternative revenue mixes (in units) that have a contribution margin of $2,000 and thus result in breakeven operations include the following:

Do-All	25	20	15	10	5	0
Superword	0	10	20	30	40	50
Total	25	30	35	40	45	50

Exercises

Wembley Travel is a travel agency specializing in flights between Toronto and London. It books passengers on Canadian Airlines. United charges passengers $900 per round-trip ticket. Until last month, Canadian Airlines paid Wembley a commission of 10% of the ticket price paid by each passenger. This was Wembley's only source of revenues. Wembley's fixed costs are $14,000 per month (for salaries, rent, etc.), and its variable costs are $20 per ticket purchased for a passenger. This $20 includes a $15 per ticket delivery fee paid to Federal Express. (To keep the analysis simple, we assume each round-trip ticket purchased is delivered in a separate package; thus the $15 delivery fee applies to every ticket.)

Canadian Airlines has just announced a revised payment schedule for travel agents. It will now pay travel agents a 10% commission per ticket up to a maximum of $50. Any ticket costing more than $500 receives only a $50 commission, irrespective of the ticket price.

REQUIRED

1. Under the old 10% commission structure, how many round-trip tickets must Wembley sell each month to (a) break even and (b) earn an operating income of $7,000 per month?
2. How does Canadian's revised payment schedule affect your answers to (a) and (b) in requirement 1?
3. Wembley is approached by DHL Express, who offers to charge $9 per ticket delivered. How would accepting this offer affect your answers to (a) and (b) in requirement 2? (Assume the maximum commission is $50 per ticket.) DHL Express offers next-day service, with reliability comparable to Federal Express.

SOLUTION

1. Wembley receives a 10% commission on each ticket—10% × $900 = $90. Thus:

$$\text{USP} = \$90$$

$$\text{UVC} = \$20$$

$$\text{UCM} = \$90 - \$20 = \$70$$

$$\text{FC} = \$14{,}000 \text{ per month}$$

a. $Q = \dfrac{\text{FC}}{\text{UCM}} = \dfrac{\$14{,}000}{\$70} = 200$ tickets per month

b. When target operating income (TOI) = $7,000 per month:

$$QT = \frac{\text{FC} + \text{TOI}}{\text{UCM}}$$

$$= \frac{\$14{,}000 + \$7{,}000}{\$70} = \frac{\$21{,}000}{\$70}$$

$$= 300 \text{ tickets per month}$$

2. Wembley receives only $50 on the $900 ticket because it exceeds $500. Thus:

$$\text{USP} = \$50$$

$$\text{UVC} = \$20$$

$$\text{UCM} = \$50 - \$20 = \$30$$

$$\text{FC} = \$14{,}000 \text{ per month}$$

a. $Q = \dfrac{\$14{,}000}{\$30} = 467$ tickets (rounded up)

b. $QT = \dfrac{\$21{,}000}{\$30} = 700$ tickets

3-20 **CVP exercises.** The Super Donut owns and operates six donut outlets in and around Quebec City. You are given the following corporate budget data for next year:

Revenues	$10,000,000
Fixed costs	1,700,000
Variable costs	8,200,000

Variable costs change with respect to the number of donuts sold.

REQUIRED

Compute the budgeted operating income for each of the following deviations from the original budget data. (Consider each case independently.)

1. A 10% increase in contribution margin, holding revenues constant
2. A 10% decrease in contribution margin, holding revenues constant
3. A 5% increase in fixed costs
4. A 5% decrease in fixed costs
5. An 8% increase in units sold
6. An 8% decrease in units sold
7. A 10% increase in fixed costs and 10% increase in units sold
8. A 5% increase in fixed costs and 5% decrease in variable costs

3-21 **CVP exercises.** The Doral Company manufactures and sells pens. Present sales output is 5,000,000 units per year at a selling price of $0.50 per unit. Fixed costs are $900,000 per year. Variable costs are $0.30 per unit.

REQUIRED

(Consider each case separately.)

1. **a.** What is the present operating income for a year?
   b. What is the present breakeven point in revenues?

Compute the new operating income for each of the following changes:

2. A $0.04 per unit increase in variable costs
3. A 10% increase in fixed costs and a 10% increase in units sold
4. A 20% decrease in fixed costs, a 20% decrease in selling price, a 10% decrease in variable costs per unit, and a 40% increase in units sold

Compute the new breakeven point in units for each of the following changes:

5. A 10% increase in fixed costs
6. A 10% increase in selling price and a $20,000 increase in fixed costs

3-32 **CVP, sensitivity analysis.** Hoot Washington is the newly elected charismatic leader of the Western Party. He is the darling of the right-wing media. His "take no prisoners" attitude has left many an opponent on a talk show feeling run over by a Mack truck.

Media Publishers is negotiating to publish *Hoot's Manifesto*, a new book that promises to be an instant bestseller. The fixed costs of producing and marketing the book will be $500,000. The variable costs of producing and marketing will be $4 per book. These costs are before any payments to Hoot. Hoot negotiates an up-front payment of $3 million plus a 15% royalty rate on the net sales price of each book. The net sales price is the listed book store price of $30 minus the margin paid to the book store to sell the book. The normal book store margin of 30% of the listed book store price is expected to apply.

REQUIRED

1. Present a PV graph for Media Publishers.
2. How many copies must Media Publishers sell to (a) break even and (b) earn a target operating profit of $2 million?
3. Examine the sensitivity of the breakeven point to the following changes:
   a. Decreasing the normal bookstore margin to 20% of the listed book store price of $30
   b. Increasing the listed book store price to $40 while keeping the book store margin at 30%.

Comment on the results.

3-33 **CVP, changing inputs (continuation of 3-32).** Hoot Washington's up-front payment of $3 million attracts a lot of negative publicity. His opponents claim he is "in bed" with the right-wing owner of Media Publishers. The government is considering relaxing some key media ownership restrictions that would expand the opportunities available to many media companies. Hoot and Media Publishers decide to drop the $3 million up-front payment, but make Hoot's royalty rate 20% of the listed book store price of $30. The normal book store margin of 30% of the listed book store price is expected to apply.

REQUIRED

1. Present a PV graph for Media Publishers with the revised contract.
2. How many copies must Media Publishers sell to (a) break even and (b) earn a target operating profit of $2 million? Compare your answers with those in requirement 2 of Problem 3-32.
3. What number of copies must Media Publishers sell for Hoot to be indifferent to receiving either (a) a $3 million up-front payment and a 15% royalty rate, on net sales price or (b) no up-front payment and a 20% royalty rate on listed book store price?

3-37 **Revenue mix, three products.** The Ronowski Company has three product lines of belts, A, B, and C, with contribution margins of $3, $2, and $1 respectively. The president foresees sales of 200,000 units in the coming period, consisting of 20,000 units of A, 100,000 units of B, and 80,000 units of C. The company's fixed costs for the period are $255,000.

REQUIRED

1. What is the company breakeven point in units, assuming that the given revenue mix is maintained?
2. If the mix is maintained, what is the total contribution margin when 200,000 units are sold? What is the operating income?
3. What would operating income become if 20,000 units of A, 80,000 units of B, and 100,000 units of C were sold? What is the new breakeven point in units if these relationships persist in the next period?

3-38 **Revenue mix, three products.** The Mendez Company has three products, tote bags H, J, and K. The president plans to sell 200,000 units during the next period, consisting of 80,000 units of H, 100,000 units of J, and 20,000 units of K. The products have unit contribution margins of $2, $3, and $6 respectively. The company's fixed costs for the period are $406,000.

REQUIRED

1. Compute the budgeted operating income. Compute the breakeven point in units, assuming that the given revenue mix is maintained.
2. Suppose 80,000 units of H, 80,000 units of J, and 40,000 units of K are sold. Compute the budgeted operating income. Compute the new breakeven point in units if these relationships persist in the next period.

3-39 **Revenue mix, two products.** The Goldman Company retails two products, a standard and a deluxe version of a luggage carrier. The budgeted income statement is as follows:

	Standard Carrier	Deluxe Carrier	Total
Units sold	150,000	50,000	200,000
Revenues @ $20 and $30 per unit	$3,000,000	$1,500,000	$4,500,000
Variable costs @ $14 and $18 per unit	2,100,000	900,000	3,000,000
Contribution margins @ $6 and $12 per unit	$ 900,000	$ 600,000	1,500,000
Fixed costs			1,200,000
Operating income			$ 300,000

Requires the breakeven point in units if the mix is maintained. If onlu standard carriers are sold and, b if only deluxe carriers are sold.